

RHEODYNAMICS AND MASS TRANSFER OF A CYLINDER OSCILLATING  
IN A VISCOELASTIC FLUID

N. A. Pokryvailo, V. V. Tovchigrechko,  
Z. P. Shul'man, and T. V. Yushkina

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The rheodynamics and convective diffusion of a cylinder oscillating in a viscoelastic fluid are investigated theoretically and experimentally.

It is generally known that acoustic oscillations significantly accelerate heat- and mass-transfer processes [1] due to the fact that, along with periodic flow, a secondary steady fluid flow (streaming) is induced in the vicinity of the body by the interaction of viscous and inertial forces in the boundary layer.

A vast number of investigations of the hydrodynamics of heat and mass transfer from bodies oscillating in a Newtonian fluid have been critically reviewed in several papers [1-4].

A solution of the flow problem near an oscillating cylinder in the boundary-layer approximation under the condition  $A/d \ll 1$  was first published by Schlichting [5]. The secondary flow pattern based on this solution is illustrated in Fig. 1a.

In each quadrant we observe two zones separated by a null streamline (at which the radial component of the steady secondary streaming velocity is equal to zero) and having opposite flow directions: an inner zone (sometimes called the inner boundary layer  $\delta$ ) immediately adjacent to the cylinder, and an outer zone. The dimensions of the inner zone are exaggerated in Fig. 1a. This flow pattern is confirmed by visualization tests [5-7]. Schlichting's solution describes the velocity of the outer flow zone according to the relation

$$u = -\frac{3}{4} \frac{U_0}{\omega} \frac{dU_0}{dx},$$

but, as shown by Stuart, the configuration of the steady streaming in the outer zone depends on the quantity  $A^2 \omega / \nu = Re_b$ , which has the significance of a Reynolds number. When  $Re_b$  is small ( $Re_b < 10$ ), the flow in the outer problem is described by Schlichting's solution, and for  $Re_b > 10$  there is formed in the outer flow a boundary layer, at whose outer boundary  $u = 0$  [8].

Investigations of the processes of mass transfer toward an oscillating cylinder are also based on boundary-layer theory. It has been shown [9, 10] that the time-varying diffusion flux is small in comparison with the constant flow and the mass transfer is determined by the secondary steady streaming.

The diffusion Prandtl numbers are large, so that the problem is usually simplified, the condition of small thickness of the diffusion boundary layer in comparison with the hydrodynamic boundary layer can be used, and the velocity profile can be linearized. However, in the case of acoustic streaming the ratio of the layer thickness has the form [2, 3]

$\frac{\delta_D}{\delta} = \frac{d}{A} Pr^{-1/2}$  and depends only on the number  $Pr$ . The condition  $\delta \gg \delta_D$  holds for not too

small amplitudes or for very large numbers  $Pr$ .

If the amplitude is very small, a situation can arise in which  $\delta_D \gg \delta$ , but  $\delta_D \ll d$ . In the first limiting case the diffusion is determined by the inner flow zone, and in the second limit by the outer zone. For  $\delta_D \gg \delta$  the value of the number  $Re_b$  is significant for the determination of the velocity field.

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A. V. Lykov Institute of Heat and Mass Transfer, Academy of Sciences of the Belorussian SSR, Minsk. Translated from *Inzhenerno-Fizicheskii Zhurnal*, Vol. 38, No. 5, pp. 820-828, May, 1980. Original article submitted February 9, 1979.

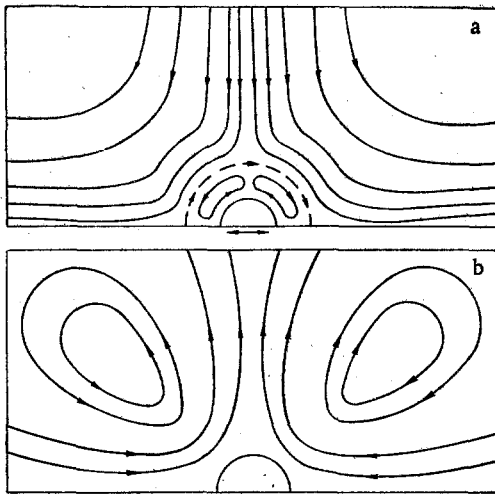


Fig. 1. Steady secondary streaming patterns. a) Newtonian fluid: 50% solution of glycerin in water; b) POE solution.

In the limit  $Pr \rightarrow \infty$  a solution of the diffusion problem for an oscillating cylinder has been obtained in [11, 9, 12]:

$$I = 4bR \int_0^{\pi/2} j d\theta = 3,28D^{2/3} C_0 R^{1/3} A^{2/3} \omega^{1/2} \nu^{-1/6} b, \quad (1)$$

$$Nu = \frac{I2R}{2\pi R b D C_0} = 0,826 Pr^{1/3} Re^{1/2} \left( \frac{A}{d} \right)^{1/6} \quad (2)$$

subject to the condition that

$$\frac{A}{d} \ll 1, \quad \frac{A}{d} Pr^{1/2} \gg 1. \quad (3)$$

The case in which the mass transfer toward the cylinder is determined by the outer secondary zone has been investigated for  $Re_b < 10$  in [11, 9, 13] and for  $Re_b > 10$  in [11]. The diffusion problem for an oscillating sphere has been solved in [14].

It is well known that the addition of a small quantity of certain polymers (as a rule, polyoxyethylene—POE) to a Newtonian fluid endows the fluid with viscoelastic properties and changes the hydrodynamic pattern, thereby affecting, in turn, the nature of the mass-transfer process between the surface and the medium.

The secondary streaming of a viscoelastic fluid around a cylinder oscillating in a direction perpendicular to its axis has been visualized [15] for one oscillatory regime  $A/d = 0.25$ ,  $f = 45$  Hz and one concentration of Dow Separan AP 30 ( $C = 100$  ppm); it is shown in this work that the inner flow zone is greatly expanded with the formation of streaming opposite to the direction for a Newtonian fluid. The reversal of secondary streaming in viscoelastic fluids has been noted in other papers [16, 17].

A solution in the boundary-layer approximation has been obtained [18] for the problem of the flow of a Walters fluid B' [19] with small relaxation times (i.e., considerably smaller than the oscillation period) around an oscillating cylinder. The equation of state in this case has the form [19]

$$P^{ik} = 2\eta_0 e^{(1)ik} - 2k_0 \frac{\delta e^{(1)ik}}{\delta t}, \quad (4)$$

$$\eta_0 = \int_0^{\infty} N(\tau) d\tau, \quad k_0 = \int_0^{\infty} \tau N(\tau) d\tau,$$

$$\frac{\delta b^{ik}}{\delta t} = \frac{\partial b^{ik}}{\partial t} + v^m \frac{\partial b^{ik}}{\partial x^m} b^{im} - \frac{\partial v^i}{\partial x^m} b^{mk}, \quad (5)$$

where  $N(\tau)$  is the relaxation-time distribution function. For a Newtonian fluid  $N(\tau) = \eta_0 \delta(\tau)$ .

An analysis of the solution in [18] indicates expansion of the inner secondary flow zone with strengthening of the viscoelastic properties, a result that is qualitatively consistent with the data of [15], along with an increase in the flow rate.

It is important to note that the zeroth approximation of the solution in [18] (particle velocity), unlike (for example) [20], does not depend on the viscoelastic properties of the fluid and corresponds exactly with the zeroth approximation of Schlichting's solution [5]. The reason for this agreement is that the author of [18] omits the term  $(k_0/\rho) (\partial^3 u / \partial t \partial y^2)$  in the boundary-layer equation, and according to our estimates such an omission is unjustified. The retention of this term complicates the solution procedure, but yields results different from [18].

The unsteady boundary-layer equation for the fluid (4), (5) has the form [21]

$$\begin{aligned} \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial P}{\partial x} + \frac{\eta_0}{\rho} \frac{\partial^2 u}{\partial y^2} \\ - \frac{k_0}{\rho} \left[ \frac{\partial^3 u}{\partial t \partial y^2} + u \frac{\partial^3 u}{\partial x \partial y^2} + v \frac{\partial^3 u}{\partial y^3} + \frac{\partial u}{\partial x} \frac{\partial^2 u}{\partial y^2} - \frac{\partial u}{\partial y} \frac{\partial^2 u}{\partial x \partial y} \right]; \\ -\frac{1}{\rho} \frac{\partial P}{\partial x} = \frac{\partial U}{\partial t} + U \frac{\partial U}{\partial x}; \quad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \end{aligned} \quad (6)$$

where

$$U = U_0(x) \exp(i\omega t). \quad (7)$$

Let us assume that the amplitude of the oscillations is much smaller than the diameter of the cylinder. We seek a solution in the form [5]

$$u(x, y, t) = u_0(x, y, t) + u_1(x, y, t) \quad (8)$$

subject to the boundary conditions

$$u_0 = u_1 = 0, \quad y = 0, \quad u_1 = 0, \quad u_0 = U_0(x) \exp(i\omega t). \quad (9)$$

Then

$$\begin{aligned} u_0 = U(x) \left[ 1 - \exp\left(-\sqrt{\frac{i}{1-ki}} \eta\right) \exp(i\omega t) \right], \\ \eta = y \sqrt{\frac{\omega}{\nu}}; \quad \nu = \frac{\eta_0}{\rho}. \end{aligned} \quad (10)$$

and the quantity of interest, namely the steady-flow stream function, has the form

$$\begin{aligned} f'_{1b} = \frac{1}{2} \left\{ 2 \left[ \frac{1}{a^2 + b^2} \left( 2 \frac{a^2 - b^2}{a^2 + b^2} - \frac{1}{2} - \frac{k}{2} (a^2 - b^2) \right) + 2k \right] \right. \\ \left. - 2 \left\{ \left[ \frac{1}{a^2 + b^2} \left( 2 \frac{a^2 - b^2}{a^2 + b^2} - \frac{1}{2} - \frac{k}{2} (a^2 - b^2) \right) + 2k \right] \cos b\eta \right. \right. \\ \left. \left. - \left[ \frac{2ab}{a^2 + b^2} \left( \frac{2}{a^2 + b^2} + \frac{k}{2} \right) \right] \sin b\eta \right\} \exp(-a\eta) \right. \\ \left. - \left[ \left( \frac{a}{a^2 + b^2} + ak \right) \cos b\eta + \left( bk - \frac{b}{a^2 + b^2} \right) \sin b\eta \right] \eta \exp(-a\eta) \right. \\ \left. + \left[ -1 + \frac{a^2 - b^2}{a^2 + b^2} - 2k(a^2 - b^2) + k(a^2 + b^2) \right. \right. \\ \left. \left. + k \left( \frac{(a^2 - b^2)^2}{a^2 + b^2} - \frac{4a^2 b^2}{a^2 + b^2} \right) \right] \frac{1}{4a^2} [1 - \exp(-2a\eta)] \right\}, \end{aligned} \quad (11)$$

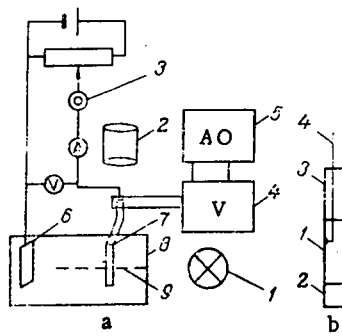


Fig. 2. Experimental circuit. a: 1) Strobe light source; 2) microscope; 3) loop oscillograph; 4) vibrator; 5) audio oscillator; 6) anode; 7) cylindrical cathode; 8) cell; 9) visualization plane; b: 1) platinum tube; 2, 3) insulators; 4) conductor.

where

$$a^2 = \frac{1}{2(k^2 + 1)} [(k^2 + 1)^{1/2} - k]; \quad b^2 = \frac{[(k^2 + 1)^{1/2} + k]}{2(k^2 + 1)}; \quad k = \frac{k_0 \omega}{\eta_0}. \quad (12)$$

Expression (11) differs from the corresponding relation (24) in [18].

For a Newtonian fluid the solution obtained here goes over to Schlichting's solution.

Calculations according to (11) show that with strengthening of the viscoelastic properties of the fluid the inner secondary streaming zone expands, and the flow rate increases. The increased secondary flow rate should probably cause the mass-transfer process to be intensified.

The solution of the convective diffusion problem

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2}, \quad C = C_0, \quad y = 0, \quad C = 0, \quad y \rightarrow \infty$$

for a cylindrical electrode oscillating in a viscoelastic fluid (4), (5), being determined by the secondary steady streaming (11), yields an expression for the number Nu, constructed with respect to the diameter, under the condition  $Pr \gg 1$ :

$$Nu = 0.814 Re^{1/2} Pr^{1/3} \left( \frac{A}{d} \right)^{1/6} [2\sqrt{2} f_{1b}''(0)]^{1/3}, \quad (13)$$

$$Re = \frac{A\omega d}{\nu}, \quad \frac{A}{d} \ll 1, \quad \frac{A}{d} Pr^{1/2} \gg 1,$$

$$f_{1b}'' = \frac{b}{(k^2 + 1)^{1/2}} \left[ 2(k^2 + 1)^{1/2} + \frac{k}{2} \right] + a \left[ \frac{k^2}{2} \frac{1}{(k^2 + 1)^{1/2}} - (k^2 + 1) - \frac{k}{2} \right] + \frac{1}{4a} \left[ -1 + \frac{2k^2}{k^2 + 1} + \frac{k^3}{(k^2 + 1)^{3/2}} - \frac{k}{(k^2 + 1)^{1/2}} \right]. \quad (14)$$

For  $k = 0$  relation (14) takes the form

$$f_{1b}''(0) = \frac{1}{2\sqrt{2}}, \quad (15)$$

and  $f_{1b}''(0)$  increases with the value of  $k$ . For  $k \ll 1$  expression (14) can be simplified:

$$f_{1b}''(0) = \frac{1}{\sqrt{2}} \left( \frac{1}{2} + \frac{3}{4} k \right). \quad (16)$$

All other conditions being equal, we infer from (13)-(16) that the mass-transfer rate depends on the value of  $k$ , the influence of the viscoelastic properties increasing with the oscillation frequency  $f$ .

The experimental arrangement for visualization of the flow around an oscillating cylinder is shown schematically in Fig. 2a. The cylinder 7 with a diameter of 1 mm and length of 40 mm is attached to the holder of the vibrator 4. The frequency and amplitude are set by the audio oscillator 5. The investigated fluid is placed in the cell 8, which has dimensions 55 × 80 × 55 mm. Magnesium oxide particles with diameters of 10-30 μm are used for flow visualization. The secondary streaming zones are clearly visible only with the use of a strobe light source. A slit is placed in front of the source for delineation of the visualization plane, which is represented by a dashed line in the figure.

Visualization is performed in a Newtonian fluid comprising a 50% solution of glycerin in water and in viscoelastic fluids obtained by the addition of polyoxyethylene WSR-301 to the glycerin-water solution.

The use of a glycerin-water solution expands the dimensions of the inner zones in comparison with pure water, thereby facilitating visualization; also, the variation of the viscosity due to the WSR-301 additives is small in comparison with the viscosity of the glycerin-water solution, making it possible to isolate the actual influence of the viscoelastic properties on the flow pattern.

Unlike [15], visualization is performed in POE solutions of different concentrations (5, 10, 40, and 100 ppm) and for different oscillation amplitudes and frequencies ( $A = 0.05-0.6$  mm;  $f = 50, 70$  Hz).

In general, two streaming zones, an inner and an outer, are observed in the solution without the polymer under all conditions (Fig. 1a). A difference shows up only with increasing external velocity in the dimensions of the inner zones and in the intensities of the outer and inner streaming.

The viscoelastic properties of the solutions alter the observed pattern.

The streaming zone adjacent to the cylinder, as in [15], expands, qualitatively confirming the theoretical analysis, and the flow pattern has the form shown in Fig. 1b. Thus, only one zone is observed, in which the direction of flow coincides with the direction in the inner zone of a Newtonian fluid.

The outer, very slow streaming zone appears in the field of view of the investigated regimes only in a 5 ppm solution at an amplitude of 0.4 mm and frequency of 50 Hz and at an amplitude of 0.3 mm and  $f = 70$  Hz. The flow pattern in this case is qualitatively similar to streaming in a Newtonian fluid.

Moreover, at a frequency of 70 Hz and amplitudes of 0.4-0.5 mm the described flow pattern changes at all concentrations. It first becomes unstable, loses symmetry, and then settles into an asymmetric streaming different from the small-amplitude case. At a frequency of 50 Hz this effect is also observed, but only in a 100 ppm solution (the visualization of streaming with large amplitudes will be described in subsequent papers).

The total mass transfer was studied experimentally by the electrodiffusion method [22] on the arrangement shown in Fig. 2. The cylinder functions as the cathode. An N-700 loop oscillograph is added to the usual circuit for the recording of unsteady diffusion flows. Figure 2b gives the structure of the cylindrical electrode ( $d = 1$  mm). The reacting surface is the platinum tube 1, which has a diameter of 1 mm and length of 5 mm. The insulator 2 of length 3 mm is used to eliminate end effects. The total length of the cylinder is 16 mm. The vibrator consists of an electromagnetic driver system and armature. The transducer holder is attached to the armature. The oscillation frequency is set by the audio oscillator in the range from 20 to 100 Hz. The oscillation amplitude is recorded by the microscope with a scale division of 10 μm. The electrolyte in this case is an ( $12.5 \cdot 10^{-4}$  M) aqueous solution of ferroferrocyanides  $K_4Fe(CN)_6/K_3Fe(CN)_6$  with  $K_2SO_4$  supporting electrolyte. The working liquids are the pure electrolyte and a solution of polyoxyethylene WSR-301 with a concentration of 100 ppm in the electrolyte.

The viscosity of the solution is measured on a capillary viscometer in the range from 1 to 1300  $sec^{-1}$ , corresponding to the experimentally observed shear velocities. In the investigated shear velocity range the viscosity is constant and equal to 0.0119 P. The coefficient of diffusion of  $Fe(CN)_6^{3-}$  ions is measured by the surge-current method.

The experimental data are processed in the form of the relation

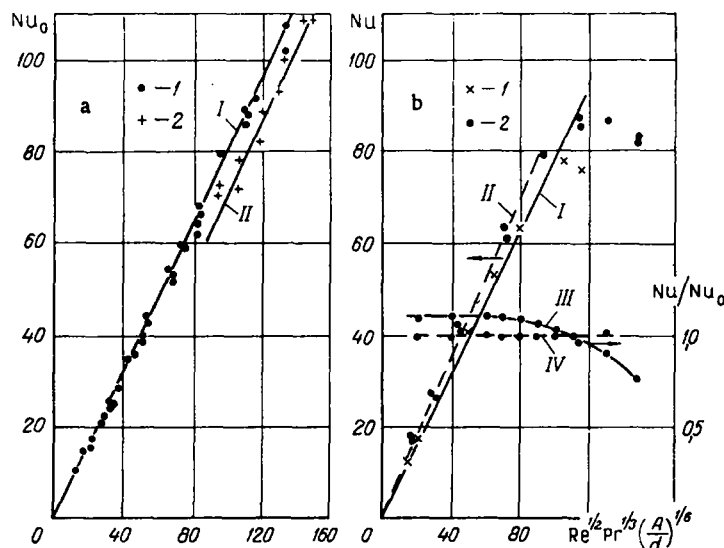


Fig. 3. Convective mass transfer of a cylinder. a) Electrolyte solution: 1) experimental data with amplitudes of 0.03-0.4 mm; 2) the same, 0.5-0.7 mm; I) calculated according to (2); II) approximation of data for  $A = 0.5-0.7$ . b) WSR-301 solution with concentration of 100 ppm in the electrolyte: 1)  $f = 30$  Hz,  $A = 0.03-0.7$  mm; 2)  $f = 60$  Hz,  $A = 0.03-0.6$  mm; I) calculated according to (2); II) according to (13); III) ratio of mass-transfer coefficient in WSR-301 polymer solution (100 ppm) to mass-transfer coefficient in solvent; IV) mass-transfer data for degraded WSR-301 solution (100 ppm).

$$Nu = f \left( Re^{1/2} Pr^{1/3} \left( \frac{A}{d} \right)^{1/6} \right).$$

The measurements are carried out with amplitudes of 0.03-0.7 mm and frequencies  $f = 30, 60,$  and 80 Hz.

It is essential to note that difficulties arise in the investigation of the total mass transfer governed by inner streaming due to the "closure" of its lines at the null streamline.

If the diffusion current is measured by the usual procedure, i.e., at a certain time after the electrode is switched into the circuit, strong scatter takes place, and the experimental data fall below the asymptotic relation (2). Such a deviation from the theory and strong scatter of the experimental data have also been observed in [14, 23]. The reason for this lowering of the current could be the gradual reduction of the concentration of material reacting at the cathode due to very low-intensity transfer to the outer streaming separated by the null streamline.

To eliminate the influence of closure of the inner streaming the measurements are carried out by an unsteady method [24]. The diffusion current is recorded on an oscillogram, and the values of the current are determined immediately after stabilization of the steady state. The results obtained in this way exhibit good reproducibility of the experimental mass-transfer data, which practically coincide for the electrolyte solution without additives, as is evident from Fig. 3a (curve I), with the analytical (2). Experimental points approximated by relation (2) are obtained in the amplitude range from 0.03 to 0.4 mm. For amplitudes of 0.5-0.7 mm the approximation is different (curve II in Fig. 3a).

The results of the investigation of mass transfer in the WSR-301 polymer solution exhibit the theoretically expected increase of the current in comparison with the electrolyte without additives, where the increase is greater the higher the oscillation frequency.

Figure 3b gives the experimental mass-transfer data for the WSR-301 solution (100 ppm) for  $f = 30, 60$  Hz and  $A = 0.03-0.7$  mm.

An increase in the mass-transfer rate in the polymer solution relative to the pure solvent by 6-7% at  $f = 30$  Hz and by 18-20% at  $f = 60$  Hz is observed up to a certain value of the group  $Re^{1/2} Pr^{1/3} \times (A/d)^{1/6}$ . With an increase in the amplitude the current values approach the points for the solvent, and with a further increase in  $A$  a rather abrupt drop in the currents in the polymer solution sets in (curve III in Fig. 3b).

This behavior of the mass transfer in the polymer solution is probably attributable to restructuring of the flow at large amplitudes. (The results of the mass-transfer investigation for large oscillation amplitude require special treatment, which is beyond the scope of the present article.)

Curve II in Fig. 3b corresponds to calculations according to (13)-(16) for  $f = 60$  Hz under the condition that the polymer solution has a single relaxation time  $\tau = 0.7 \cdot 10^{-3}$  sec. Using the fact that, according to a number of estimates [25, 26], the relaxation time of a 100 ppm POE solution is  $10^{-3}$  sec, we may regard the agreement between the analytical and experimental as satisfactory.

Since, as was shown above, the viscoelastic properties of polymer solutions elicits a change in the flow field and mass transfer near the surface of an oscillating cylinder, it is instructive to test the influence of mechanical degradation, which induces a partial or complete loss of the viscoelastic properties of polyoxyethylene solutions. Degradation is realized by prolonged intense mixing of a 100 ppm POE solution with a blade mixer operating at  $n = 1000$  rpm.

The results of the mass-transfer measurements for a cylinder oscillating in the degraded solution at  $f = 60$  Hz are given in Fig. 3b (curve IV). It is seen that in the degraded solution the mass-transfer data practically coincide with the data obtained in the electrolyte without polymer additives.

#### NOTATION

$A$ , amplitude;  $d$ , diameter;  $R$ , radius;  $x, y$ , coordinates;  $t$ , time;  $f$ , frequency;  $C_0$ , concentration in flow;  $D$ , diffusion coefficient;  $\omega$ , cyclic frequency;  $\delta(\tau)$ , delta function.

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#### HYDRODYNAMICS OF A FLUIDIZED BED IN THE INTERTUBE SPACE OF STAGGERED AND IN-LINE TUBE BUNDLES

V. N. Korolev and N. I. Syromyatnikov

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The article presents the results of the experimental investigation of the effect of the horizontal and vertical pitches of tubes on the hydrodynamics of a fluidized bed. A formula is presented for calculating the mean porosity of the bed in the intertube space of tube bundles with optimum arrangement.

One of the most promising recent trends in improving boilers is the low-temperature combustion of solid fuels in a fluidized bed by removing the heat from the combustion zone with the aid of cooling surfaces made in the form of horizontal tube bundles in staggered or in-line arrangement. The results of investigations of the gasification and combustion of solid fuel in suspended state [1] indicate that the zone of active combustion in such systems does not exceed 200 mm from the level of the gas-distributing grid. Therefore, if heat-exchange surfaces are to be situated in such a zone to ensure low-temperature combustion of the fuel, the surfaces have to be compact and optimally arranged.

The object of the present work is to investigate the hydrodynamics and the structure of

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S. M. Kirov Urals Polytechnic Institute, Sverdlovsk. Translated from *Inzhenerno-Fizicheskii Zhurnal*, Vol. 38, No. 5, pp. 829-835, May, 1980. Original article submitted April 2, 1979.